"Fastener Troubles, Causes & Solutions" Series **Resonance Phenomena & Bolt Fracture: Chattering Stops by Fastening Bolt Tightly?**

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1. Introduction

A variety of fractures could occur in bolted joints. When bolted joints are subjected to vibration forces and the mechanical structure resonates, it possibly causes large alternating stresses in the bolted joints or loosening of the bolts or nuts. In that case, if the bolts were tightened further firmly in the bolted joint under the state of resonance, the resonance phenomena may be stopped. This phenomenon is due to the fact that the natural frequency of the bolted joint increases as a result of raising the axial bolt force. Two kinds of phenomena are considered to be the primary cause. The first one is due to the increase in the stiffness of the bolted joint because the mating surfaces get into the complete contact condition, which was in partial contact condition under low axial force. This problem does not occur if the corresponding surfaces are in completely parallel alignment. The second phenomenon is considered to be the effect of interface stiffness. Regarding the interface stiffness, I have given a brief explanation in my sixth article, "Angle Control Method: Combining with Torque Method to Improve Fastening Precision", which appeared in p.352 of Fastener World Magazine No. 160. In order to provide some hints for preventing the troubles occurring in bolted joints subjected to vibration forces, the resonance phenomena and natural frequencies of bolted joints will be explained in this article.



2. Axial Bolt Force and Natural Frequency

The term natural frequency refers to the vibration frequency of an object when it is vibrated under free conditions. If the frequency of external force coincides with the natural frequency, the amplitude increases. This is referred to as resonance. Resonance is an extremely dangerous phenomenon for mechanical structures. In order to avoid the resonance, it is necessary to know the basic characteristics of the natural frequencies of bolted joints. Figure 1 shows the relationship between natural frequency and axial bolt force when a bolted joint vibrates under bending force. The figure also shows the natural frequency of the first mode of a cantilever beam whose thickness is equal to that of two fastened plates. The first mode indicates the vibration mode in which the beam simply deforms up and down. Additionally, in the second mode, the cantilever deforms in the form like one period of sine wave. Natural frequencies of a cantilever beam can be calculated by the following expression.

$$f = \frac{\lambda_i^2}{2\pi L^2} \sqrt{\frac{EI}{\rho A}} \quad (1)$$

Here, λi is the eigenvalue of the ith mode. L, I, and A represent the length, moment of inertia of area, and the cross sectional area of the beam, respectively. E and p are Young's modulus and density of the beam material. As is obvious from the above expression, infinitely great number of natural frequencies f exists theoretically. However, judging from the relationship with the frequency of the external force, only some frequencies become an issue in actual structures. As shown in Figure 1, the natural frequency of bolted joints changes with the magnitude of axial force. The degree of change is dependent on the conditions of contact surfaces. In this example, the target contact surfaces are the interface between two fastened objects and the bearing surfaces of the nut and the bolt head. Particularly, the interface between the fastened objects is under low pressure because of its wide area. As the axial force is increased, therefore, plastic deformations of the small asperities tend to progress, which makes the stiffness prone to increase.

Fig. 2(a) and (b) show the measurement results of natural frequencies for the two plates (3.5mm thick and 30mm width) clamped with a set of M12 bolt and nut. The former figure represents the results for the plate of 400mm length and the latter represents those for the plate of 100mm length. For measuring the natural frequencies, strain gages are attached to the surfaces near the fixed portion of the plates, and then the natural frequencies are obtained from the number of changes in the sign of strains while the plates are vibrated vertically. Two positions are chosen to place the bolt, one of which is near the free end of the plates (model A) and the other at the center portion of the plates (model B). Also, the surface roughness of the interface is varied in two levels, i.e., fine surface (f) and rather rough surface (r). According to the figure, in both cases, the natural frequencies increase with the increase of axial bolt force, and they tend to saturate as the axial bolt force reaches a certain extent. Additionally, compared to model A, model B has wholly higher natural frequencies. Regarding to surface roughness, natural frequencies tend to decrease with the increase of surface roughness.

Natural frequencies of a 7mm thick beam, calculated using Eq.1, are marked in the figures. This value can be regarded as a theoretical value for the two plates being completely stuck together by ignoring the increase of mass owing to the use of bolts and nuts. All the natural frequencies obtained from the experiments are lower than the theoretical value of the beam. The reason is that the range of contact surface under contact pressure caused by axial bolt force is restricted to the interface surrounding the bolt.

3. Analysis of Natural Frequency by **Taking Account of Interface Stiffness**

If the experimental results shown in Figure 2 can be reproduced through numerical analysis, it becomes possible to evaluate natural frequencies of bolted joints with various forms by taking the effects of axial bolt force and surface roughness into consideration. For that purpose, it is necessary to derive an expression relating to surface roughness and interface stiffness. Figure 3 is an exaggerated shape of contact surfaces. If the mating surfaces were subjected to compression force in the initial state, small asperities on the surface deform plastically and the two contact surfaces would come closer between each other. The amount of that deformation is called approach of interface ζ , which becomes a large values even under low contact pressure p. Ostrovskii proposed the following expression for the relationship between the approach of interface ζ and contact pressure p;

 $\zeta = cp^m$ (2)

where c and m are the constants obtained from experiments. These constants can be expressed as a function of Rzt, which is the sum of the maximal height roughness of corresponding two contact surfaces.

$$c = 0.0674R_{zt} + 0.413$$

$$m = 0.0115R_{zt} + 0.155$$
 (3)

As is obvious from Expression 2, the approach of interface ζ changes nonlinearly with contact pressure p, which means that the interface stiffness also behaves nonlinearly with contact pressure. In other words, the interface stiffness increases with larger axial bolt force because of the approach of the two mating surfaces. However, the interface stiffness is not in direct proportion to axial bolt force. This is consistent with the tendency observed in the experimental results shown in Figure 2. Using Expression 2 as a fundamental equation and introducing the proposals of other researchers, the interface stiffness in the normal and tangential directions on the interface can be expressed by two nonlinear spring constants: kn and kt. Please refer to the references of this article for more detailed explanation.

Figure 4 shows an example of finite element models. The center portion of the plate corresponds to the interface, where the mating nodes are connected by the springs with spring constants of kn and kt. In this analytical model, thread surface and bearing surfaces of nut and bolt head are not modeled because their effects are considered to be relatively small. Figure 5 shows an example of analytical results for the case of the plate with 400mm length. For comparison, the experimental results are also depicted in the figure. In the analyses, Rzt was set to be 30µm for rough surface and to be 5µm for smooth surface. Although there was a slightly larger difference between the experimental and analytical values in the range of small axial bolt forces, it can be judged that the analyses of natural frequencies have been performed with sufficient accuracy. Figure 6 shows the analytical results for the case of the plate with 200mm length, which shows the same tendency as that in Figure 5. Additionally, as the axial bolt force approaches zero, the natural frequency approaches half the value of an one-body beam. This phenomenon was also observed in Figure 5, and it proves the fact that "natural frequency is in direct proportion to the beam height", suggested by Expression 1. Namely, it can be explained that as the axial force decreases, the natural frequency approaches that of each single plate.

4. Conclusion

Many bolts are used in machines and structures, and it is known that their natural frequencies change with the magnitude of axial bolt force. On the other hand, when targeting the same machines and structures, numerical analyses of vibration problems are generally harder than stress analysis. The reason is the change of interface stiffness. However, it seems that there are no research reports with a focus on the interface stiffness and surface roughness. As an extension of a daily experience of a phenomenon that resonance can be stopped by firmly tightening bolts, this article was intended to explain the resonance phenomena causing various problems in bolted joints and its natural frequencies. The next article will explain the leakage problems caused by the oil pressure exerting on bolted joints.





Reference

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